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Cambridge IGCSE®
Chemistry
Maths Skills Workbook

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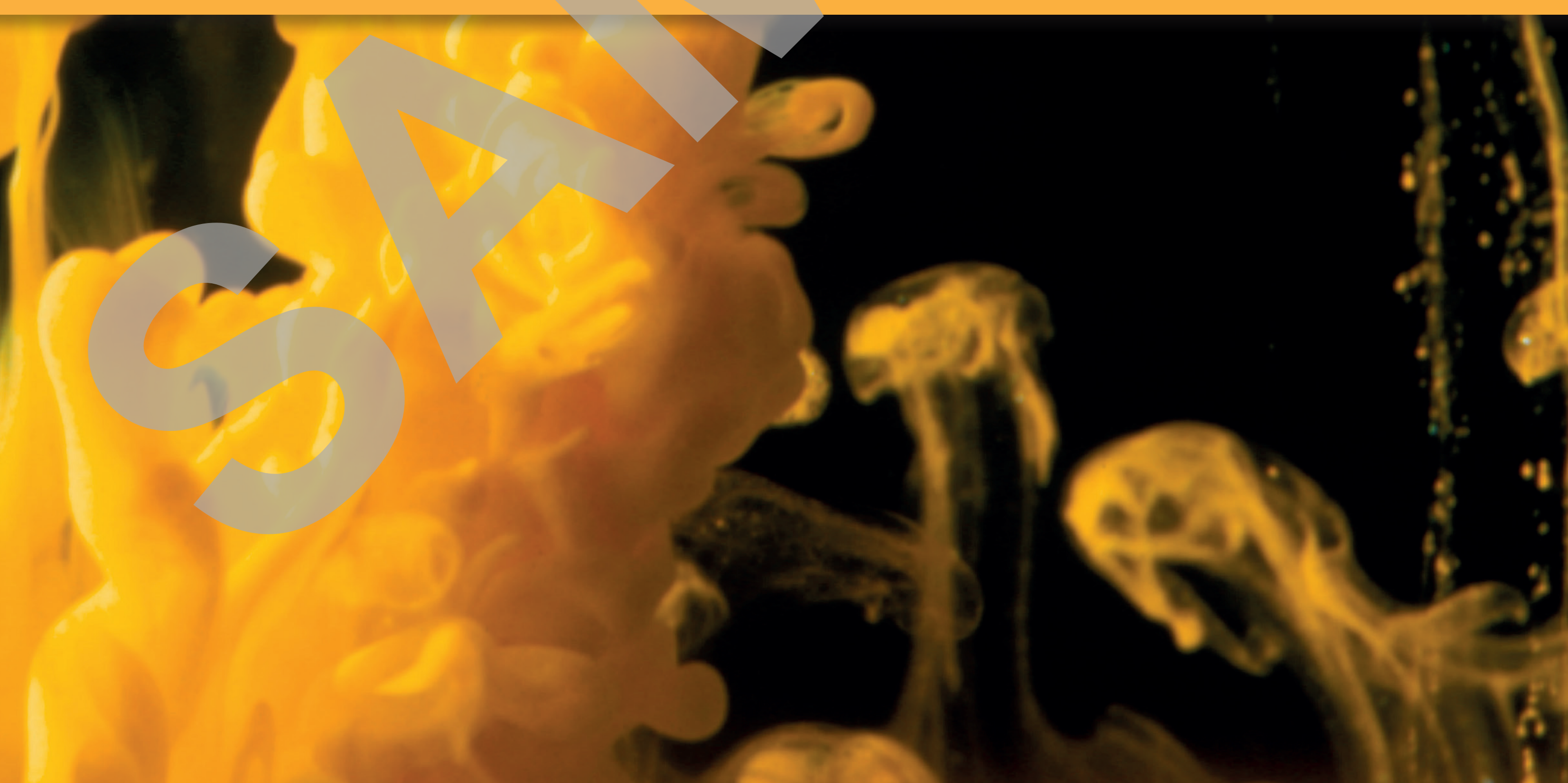


Contents

Introduction

This workbook has been written to help you to improve your skills in the mathematical processes that you need in your Cambridge IGCSE Chemistry course. The exercises will guide you and give you practice in:

- representing values
- working with data
- drawing charts and graphs
- interpreting data
- doing calculations
- working with shape.

Each chapter focuses on several maths skills that you need to master to be successful in your Chemistry course. It explains why you need these skills. Then, for each skill, it presents a step-by-step worked example of a question that involves the skill. This is followed by practice questions for you to try. These are not like exam questions. They are designed to develop your skills and understanding. They get increasingly challenging. Tips are often given alongside to guide you. Spaces, lines or graph grids are provided for your answers.

It is best to work through Chapters 1 and 2 early in your course, as they will help to ensure that you have a secure understanding of number and units, as well confidence in reading scales when making measurements. Chapter 3 shows you the skills you need to draw a variety of different types of chart and graph. These chapters will support you with many practical activities that you may carry out.

Chapter 4 covers the skills needed to read information from charts and graphs, as well as the specific graph skills that you will need when studying rates of reaction. Chapter 6 shows you the mathematics of surface area : volume ratio, which will help to explain why changing surface area affects the rate of reaction.

A few of the maths concepts and skills are only needed if you are following the Extended syllabus (Core plus Supplement). The headings of these sections are marked ‘Supplement’. In other areas just one or two of the practice questions may be based on the Supplement syllabus content, and these are also clearly marked. Most of these are in Chapter 5, which covers the key calculations needed in chemistry, including the use of moles (supplement only).

There are further questions at the end of each chapter for you to try, to give you more confidence in using the skills practised within the chapter. At the end of the book there are additional questions that may require any of the maths skills from all of the chapters.

You will find a copy of the Periodic Table on page 00. You will need this to look up relative atomic masses for some questions.

Important mathematical terms are printed in **bold** type and these are explained in the glossary at the back of the book.

Chapter 1: Representing values

Why do you need to represent values in chemistry?

- If you want to communicate measurements in chemistry, you will need to record values that you measure. You must make sure that another person will be able to understand that measurement, so how you represent it is important. As well as the numerical value, you must also include the correct **unit**.
- In chemistry, you will need to understand numbers that are much larger or much smaller than numbers you may be used to working with. Writing these numbers in different ways will make them easier to understand and compare.

Maths focus 1: Using units

All units of measure in general use are based upon **Standard International (SI) units**.

Table 1.1 shows some SI units that you may meet in chemistry.

Quantity	Unit	SI abbreviation
length	metre	m
mass	kilogram	kg
time	second	s
amount of substances	*moles**	*mol*

Table 1.1: SI units for common quantities

* Supplementary only

TIP

Remember that a temperature can take a negative value on the Celsius scale.

In chemistry the SI unit for temperature is the kelvin, but it may be more convenient to use the Celsius scale, on which the freezing point of water is 0 °C and the boiling point of water (at 1 atmosphere pressure) is 100 °C. This is more useful for many laboratory measurements, although the kelvin scale is used in more advanced chemistry studies. Note that a temperature difference in kelvin, such as 30 kelvin, is the same as a temperature difference of 30 degrees on the Celsius scale.

What maths skills do you need to be able to use units?

1 Choosing the correct unit	• Identify the type of quantity that the apparatus measures • Select an appropriate unit for that quantity
2 Writing the unit symbol	• Recall or look up the unit symbol • Check whether the unit requires index notation, for example, cm^2, cm^3
3 Writing symbols for derived units	• Work out how the quantity is calculated • Write the units to be consistent with the calculation

WATCH OUT

Not all values require units. Relative atomic mass gives the average mass of naturally occurring atoms of an element, based on a scale in which the carbon-12 atom has a mass of exactly 12 units. For example, the relative atomic mass of hydrogen is 1, meaning that, on average, atoms of hydrogen have a mass that is $\frac{1}{12}$ the mass of a carbon atom. This is a **ratio** and therefore it needs no units.

LINK

See Chapter 5 for more on ratios.

TIP

Make sure that the units are also appropriate to the scale on the measuring apparatus. A small beaker will not measure in litres.

Maths skills practice

How does using units help to communicate values measured during chemical reactions?

When you are carrying out experimental work in chemistry, it is essential that you use units to record and communicate any measurements you take.

For example, it is meaningless to state the volume of gas produced during a chemical reaction simply as ‘16’. Using units clearly specifies the volume measured. For example, a quantity of 16 cm^3 is completely different from a quantity of 16 litres. Similarly, recording the mass of product formed in an experiment as ‘3’ means nothing unless you add the correct units, such as grams. Remember that an amount of 3 g is a thousand times smaller than 3 kg, so it is essential to use the correct units.

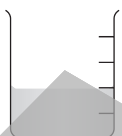
Most values used in chemistry require units as they are measures of particular quantities, such as length, mass, temperature, time, volume or the amount of a substance.

Maths skill 1: Choosing the correct unit

It is important that you can name the units commonly used in chemistry measurements.

WORKED EXAMPLE 1

Choose the correct unit of measurement associated with this small beaker.



A centimetres **B** litres **C** square centimetres **D** cubic centimetres

Step 1 Identify the type of quantity that the apparatus measures.

A beaker measures volume.

Step 2 Select an appropriate unit for that quantity.

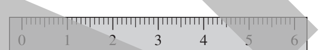
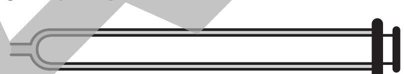
KEY QUESTIONS TO ASK YOURSELF:

- What units are used to measure this type of quantity?
  Volume may be measured in a variety of units including litres (l) or cubic centimetres (cm^3).
- Which units are appropriate for the scale on the measuring equipment?
  A small beaker will not measure litres. The scale is likely to be in cubic centimetres (cm^3).

So appropriate units in this case are cubic centimetres.

Practice question 1

Draw lines to match each item of measuring apparatus with the appropriate unit of measurement.

balance 	cubic centimetres (cm^3)
measuring cylinder 	grams (g)
thermometer 	cubic centimetres (cm^3)
ruler 	degrees Celsius (°C)
gas syringe 	centimetres (cm)

WATCH OUT

Some symbols are written starting with a capital letter. This occurs when they are named after a person who invented them, for example, the Celsius temperature is named after the Swedish astronomer Anders Celsius, who developed a similar temperature scale. Most symbols start with a lower case letter.

Maths skill 2: Writing the unit symbol

Units are not usually written out in full. Each unit has a short form, or abbreviation, comprising 1–3 letters.

Quantity	Unit	Abbreviation
length	metres	m
mass	grams	g
time	seconds	s
temperature	degrees Celsius	°C
*amount of substance**	*mole*	*mol*

Table 1.2 Abbreviations for some SI units
**Supplementary only.*

TIP

Some units are derived (worked out by calculation) from SI units.

LINK

See Chapter 5

LINK

See Maths focus 2, Maths skills 3 'Understanding unit prefixes'.

TIP

Always remember to include the correct index or **power** when necessary. It is incorrect to write a volume of liquid as 10 cm because centimetres measure length.

Other units are created by inserting a prefix in front of the SI unit. Centimetres are used for measuring shorter distances than metres, for example: 1 cm is $\frac{1}{100}$ or 0.01 m

Some units require **index** notation. For example:

- **Area** is always measured in *square* units (such as m^2 or cm^2), since it is obtained by multiplying two lengths, for example, m × m or cm × cm (think about counting squares on a grid to find areas).
- **Volume** is always measured in *cubic* units (such as m^3 or cm^3), since it is obtained by multiplying three lengths, for example, m × m × m, or cm × cm × cm (think about counting cubes in a cuboid made from unit cubes).

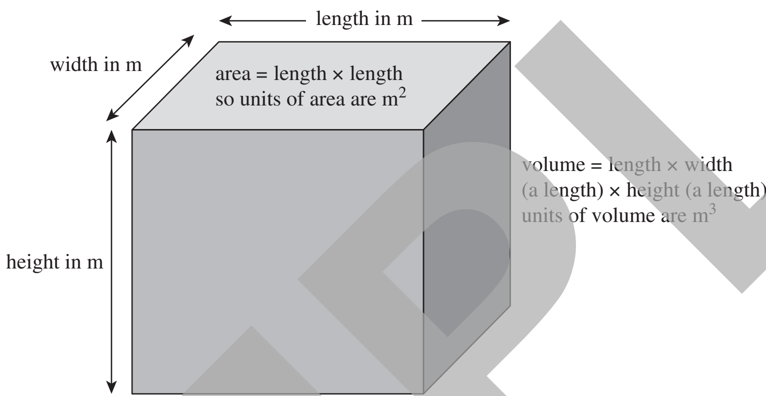


Figure 1.1 Comparing area and volume of a cuboid

WORKED EXAMPLE 2

The length and width of a piece of paper have been measured in centimetres. Write down the correct unit symbol for its area.

Step 1 Recall or look up the base unit symbol.

In this case it is centimetres (cm).

Step 2 Check whether the unit requires index notation.

Area is found by multiplying length by width so it must be measured in square units.

The unit is square centimetres (cm^2).

Practice question 2

Write down the correct unit symbol for each measurement.

a Mass of copper sulfate, measured on a digital balance that measures in grams ....

b Temperature of water, measured using a thermometer marked in degrees Celsius ....

c Time taken for a reaction to take place, measured using a stopwatch that displays seconds ....

d Length of magnesium ribbon, measured using a ruler marked in centimetres ....

e Area of floor in a laboratory, where the length and width are measured in metres ....

f Volume of liquid in a beaker that measures in cubic centimetres ....

> **TIP**
> The solidus (/) symbol indicates 'per', or division.

> **WATCH OUT**
> The symbol / is also used as a separator between a variable name and its unit, in tables and on graphs. Here, you read the / sign as 'in', so 'Temperature / °C' means 'temperature in degrees Celsius'.

Maths skill 3: Writing symbols for derived units

The units for some quantities are derived (based on a calculation) from other units. For example, you can calculate the rate of a reaction by dividing the volume of gas produced by the time taken, rather like calculating the speed of a car by dividing the distance travelled by time taken. If the volume is measured in cubic centimetres and the time in seconds, the units of rate of reaction are cm^3/s (cubic centimetres per second).

WORKED EXAMPLE 3

You can work out the density of an aluminium cube by dividing its mass (in grams) by its volume (in cubic centimetres).

Write down the correct derived unit for density.

Step 1 Work out how the quantity is calculated.

The calculation for density is: $\frac{\text{mass}}{\text{volume}}$

Step 2 Work out the derived units, to be consistent with the calculation.

The derived units are grams per cubic centimetre (g/cm³).

Practice question 3

Write down the correct derived unit for each calculated quantity:

a The rate of a reaction (how fast a reaction takes place), calculated by dividing the mass of product made (in grams) by the time taken (in seconds) ....

b The density of a bronze statue, calculated by dividing the mass of the statue (in kilograms) by its volume (in cubic metres) ....

c The rate of a reaction, calculated by dividing the volume of gas produced (in cubic centimetres) by the time taken (in seconds) ....

Maths focus 2: Understanding very large and very small numbers

In chemistry you need to understand very large numbers.

In 12 g of carbon there are about 602 000 000 000 000 000 000 000 atoms.

You also need to understand very small numbers.

A single carbon atom has a diameter of about 0.000 000 000 17 m.

It is very important to use the correct number of zeros. The value of the number depends upon the place value of the digits. If you use the wrong number of zeros, the value of the number will change.

However, writing out this many zeros takes a lot of time so very large and very small numbers are often written using **powers of ten** instead.

The number of atoms in 12 g of carbon can also be written as 6.02×10^{23}.

The diameter of a carbon atom can be written as 1.7×10^{-10} m.

Sometimes in chemistry the units are changed for very large and very small numbers by adding a **prefix** such as kilo (k) or nano (n). These prefixes replace the power of ten.

So 3 kg = 3×10^3 g or 3000 g

What maths skills do you need to be able to understand very large and very small numbers?

1 Understanding place value	• Compare digits with the highest place value • Compare digits with other place values
2 Understanding numbers expressed with powers of ten	• Write out the multiplication • Calculate the number as it would be written in full
3 Understanding unit prefixes	• Write the measurement in terms of a power of ten • Calculate the number as it would be written in full

Maths skills practice

How does understanding very large and very small numbers help to improve your understanding of the size and number of different particles?

Some numbers used in chemistry are so large, or so small, that they are difficult to imagine. Writing these in a clearer way, such as using powers of ten or prefixes, helps to understand how the size of different particles compare. A particle of PM2.5 'particulate' air pollution has a diameter of about 2.5×10^{-6} m or 2.5 µm, whereas a PM10 particle is about 10×10^{-6} m or 10 µm in diameter.

Understanding powers of ten and **unit prefixes** means that you will instantly know that these are much larger than a typical atom, which is about 1×10^{-10} m in diameter.

Before you can do this, though, it is important that you have a good understanding of place value in numbers that are written out in full.

Maths skill 1: Understanding place value

The position of a digit in a number determines its place value. The left-most digit in a number has the highest place value.

For example, the number in Table 1.3 (reading from left to right) is:

three hundred and twenty-three billion, four hundred and fifty-six million, three hundred and forty-five thousand, six hundred and forty-seven

Hundreds of billions, 10^{11}	Tens of billions, 10^{10}	Billions, 10^9	Hundreds of millions, 10^8	Tens of millions, 10^7	Millions, 10^6	Hundreds of thousands, 10^5	Tens of thousands, 10^4	Thousands, 10^3	Hundreds, 10^2	Tens, 10^1	Units, 10^0
3	2	3	4	5	6	3	4	5	6	4	7

Table 1.3 Place values for large numbers

The decimal fraction in Table 1.4 is one billionth.

		tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths	ten-millionths	hundred-millionths	billionths
0	.	0	0	0	0	0	0	0	0	1

Table 1.4 Place values for small numbers

> **TIP**
> Read the number from left to right. The place value of the first non-zero number helps you decide how big the number is.

WORKED EXAMPLE 4

Find the largest number in the following list.

A 7 242 519 **B** 8 143 921 **C** 8 349 321 **D** 924 107

Step 1 Compare digits with the highest place value.

Three of the numbers have seven digits, the fourth (D) has only six, so discard it because it is smaller.

Step 2 Compare the remaining three numbers, looking at the digits with the highest place value.

A, B and C all have millions as the highest place value. B and C both have digits showing 8 million.

Step 3 Compare digits with other place values.

The digits in B and C have the next highest place value (from left to right).

The next place value is hundreds of thousands. B has 1 hundred thousand but C has 3 hundred thousand. So the largest number is C.

Practice question 4

Circle the *largest* number in each list.

a	674 591	92 342	141 294	692 381
b	1 943 986	1 949 789	1 942 987	1 944 098
c	0.09	0.12	0.17	0.06
d	0.09	0.015	0.026	0.07
e	0.000 007 2	0.000 008 5	0.000 000 1	0.000 000 165

> **TIP**
> Look for the first non-zero digit and use place value to compare numbers.

Practice question 5

Circle the *smallest* number in each list.

a	1 232 452	123 532	723 453	115 362
b	0.123 451	0.345 984	0.135 034	0.124 093
c	0.000 002 234	0.000 002	0.000 002 4	0.000 002 34
d	234.56	234.25	232.12	232.013 4
e	104 985.99	110 374.12	104 895.99	104 895.82

Maths skill 2: Understanding powers of ten

Powers of 10 are the result of multiplying 10 by itself.

A negative power of any number is the **reciprocal** of the corresponding positive power.

This means, for example, that $10^{-1} = \frac{1}{10}$, or $1 \div 10$ (the reciprocal of 10).

$10^1 = 10$	$10^{-1} = \frac{1}{10}$ or 0.1
$10^2 = 10 \times 10 = 100$	$10^{-2} = \frac{1}{10 \times 10} = \frac{1}{100}$ or $1 \div 10 \div 10 = 0.01$
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$	$10^{-5} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{100\,000}$ $= 1 \div 10 \div 10 \div 10 \div 10 \div 10$ or 0.000 01

Table 1.5 Powers of 10

Very large and very small numbers are often recorded as multiples of powers of ten. This saves having to write out lots of zeros.

For example: $4 \times 10^3 = 4 \times 10 \times 10 \times 10 = 4000$

So multiplying by 10^3 means that you need to multiply by 10 three times.

In general 4×10^n means that 4 is multiplied by 10 n times.

4×10^1	4×10	40
4×10^2	$4 \times 10 \times 10$	400
4×10^3	$4 \times 10 \times 10 \times 10$	4000
4×10^4	$4 \times 10 \times 10 \times 10 \times 10$	40 000
4×10^5	$4 \times 10 \times 10 \times 10 \times 10 \times 10$	400 000
4×10^6	$4 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	4 000 000

Table 1.6 Multiplying by powers of 10

A negative power of any number is the reciprocal of the corresponding positive power.

For example, this means that $10^{-1} = \frac{1}{10}$, or $1 \div 10$ (the reciprocal of 10).

Multiplying a number by a negative power of ten tells you how many times to divide it by ten.

For example: $4 \times 10^{-3} = 4 \times \frac{1}{10 \times 10 \times 10} = 4 \div 10 \div 10 \div 10$

WORKED EXAMPLE 5 (POSITIVE POWERS OF TEN)

Write 5×10^5 in full.

Step 1 Write out the multiplication.

$5 \times 10^5 = 5 \times 10 \times 10 \times 10 \times 10 \times 10$

Step 2 Calculate the number as it would be written in full.

$= 5 \times 10\,000$

$= 50\,000$

Practice question 6

These numbers are expressed as multiples of powers of ten. Write them in full.

a 3×10^3 ……………………………………….

b 45×10^6 ……………………………………….

c 4×10^1 ……………………………………….

d 123×10^{10} ……………………………………….

WORKED EXAMPLE 6 (NEGATIVE POWERS OF TEN)

Write 3×10^{-4} as a decimal.

Step 1 Write out the multiplication.

$$3 \times 10^{-4} = 3 \times \frac{1}{10 \times 10 \times 10 \times 10} = 3 \div 10 \div 10 \div 10 \div 10$$

Step 2 Calculate the number as it would be written in full.

$$= 3 \times 0.0001 = 0.0003$$

Practice question 7

Write each of these negative powers of ten as decimals.

a 2×10^{-2} ……………………………………….

b 34×10^{-6} ……………………………………….

c 9×10^{-9} ……………………………………….

d 43×10^{-5} ……………………………………….

Maths skill 3: Understanding unit prefixes

Very often in science rather than writing a number either in full or using powers of ten, you can just change the unit by using a prefix.

The prefix tells you the power of ten by which to multiply the measurement to find the full number.

For example: 7 kg means $7 \times 10^3 = 7000$ g

Table 1.7 shows some prefixes you should know.

Unit prefix	Unit prefix symbol	Multiplying factor	Example unit names	Example unit symbols
kilo-	k	10^3	kilogram	kg
deci-	d	10^{-1}	cubic decimetre	dm^3
centi-	c	10^{-2}	cubic centimetre	cm^3
milli-	m	10^{-3}	milligram millimetre millilitre	mg mm ml
micro-	µ	10^{-6}	microgram	mg
nano-	n	10^{-9}	nanometre	nm

Table 1.7 Prefixes used with common measures

LINK
See Maths skill 2 'Understanding powers of ten'.

WORKED EXAMPLE 7

Write 8 mg without using the prefix.

Step 1 Write the number in terms of a power of ten.

$8\,mg = 8 \times 10^{-3}\,g$

Step 2 Calculate the number as it would be written in full.

$$8 \times 10^{-3} = 8 \times \frac{1}{10 \times 10 \times 10} = 8 \div 10 \div 10 \div 10 = 0.008$$

So $8\,mg = 8 \times 10^{-3}\,g = 0.008\,g$

Practice question 8

Write each measurement without the prefix.

a **i** 3 mg ....
ii 4 µg ....
iii 3 kg ....

b **i** 4 mm ....
ii 2 cm ....
iii 7 nm ....

c **i** 4 cm ....
ii 2 dm ....

Practice question 9

Write each measurement without the prefix.

a **i** 42 mg ....
ii 402 µg ....
iii 345 kg ....

b **i** 74 nm ....
ii 7.4 nm ....
iii 704 nm ....

Maths focus 3: Writing numbers in a required form

Sometimes in chemistry you are required to write a number in a particular form.

When very large or very small numbers are expressed in terms of a power of ten, the convention is to use a system called **standard form** or **standard index form.**

A number in standard form is expressed as a number greater or equal to one but less than 10 multiplied by a power of ten. For example, 54 000 can be written as 5.4×10^4. However, 54×10^3 is *not* in standard form because 54 is not between 1 and 10.

The results of calculations should be rounded to an appropriate number of significant figures, based upon the lowest number of significant figures of the numbers used in the calculation.

What maths skills do you need to be able to understand very large and very small numbers?

1	Writing numbers in standard form	• Rewrite the digits as a number that is greater or equal to one and less than ten • Work out how many times you have to multiply or divide the number by ten to get back to your original number
2	Writing numbers to the required number of significant figures (sf)	• Identify the correct number of significant figures • Decide whether to round up or down.

Maths skills practice

How does writing numbers in a required form help communicate chemistry?

Standard form provides a consistent system for communicating and comparing very small and large numbers. The power of ten gives a useful estimate of the size of the number.

It is important that all values in chemistry are recorded to an appropriate number of **significant figures**. Writing the result of a calculation as 34.938 475 cm^3 when in reality the measurements used in the calculation were only to three significant figures suggests a much greater degree of accuracy than actually achieved in the experiment. It is better to write the number to three significant figures, that is 34.9 cm^3.

TIP
In standard form the decimal place always come after the most significant figure.

LINK
See Maths skill 2, 'Writing numbers to the required numbers of significant figures (sf).'

Maths skill 1: Writing numbers in standard form

In general terms, a number in standard form should always include a number that is greater than or equal to one and less than ten that is then multiplied by a power of ten.

So for the number 4 060 000:

- 4.06×10^6 is correctly in standard form because 4.06 is between 1 and 10.
- 406×10^4 is in index form, but is *not* correctly in standard form because 406 is greater than 10.

TIP
Note that calculators differ, so you need to know the correct key on the calculator that you use.

Standard form on your calculator

Calculators do not all work in the same way, so you must make sure you know how to use yours. This is particularly important when you need to enter or read numbers in standard form. This may involve using the e key (or the [EE] key).

For example, to enter 1.67×10^{11}, a typical key sequence would be:

1.67 e11

The screen would show the number as:

1.67×10^{11}

WORKED EXAMPLE 8

12 g of carbon contains 6.02×10^{23} carbon atoms.

How many carbon atoms are there in 24 g of carbon?

$2 \times 6.02 \times 10^{23} = 1.2 \times 10^{24}$

WORKED EXAMPLE 9 (LARGE NUMBERS)

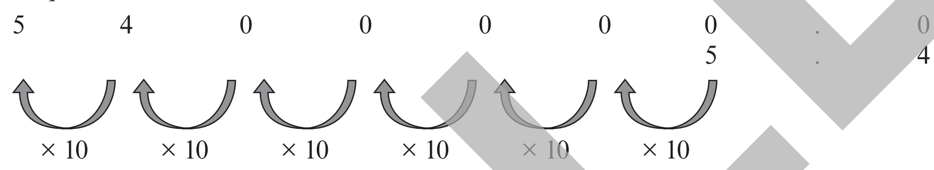
Write 5 400 000 in standard form.

Step 1 Write the digits as a number that is greater than or equal to one and less than ten.

5.4

Step 2 Work out how many times you have to multiply this number by ten to get back to your original number.

5.4 must be multiplied by 10 six times ($5.4 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$) *to equal 5 400 000.*



Step 3 Write the number, using the correct power of ten.

$5.4 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 5 \times 10^6$

TIP

If there are significant zeros between the digits in a number, these must appear in the number written in standard form, for example: $3\,050\,000 = 3.05 \times 10^6$

Practice question 10

Write these numbers in standard form.

a 134 000 ....

b 103 000 ....

c 120 000 000 ....

d 140 ....

Practice question 11

Write these values in standard form.

a 34 000 000 000 000 carbon atoms ....

b 142 000 g ....

c 145 m^3 ....

The method for converting very small numbers into standard form is slightly different.

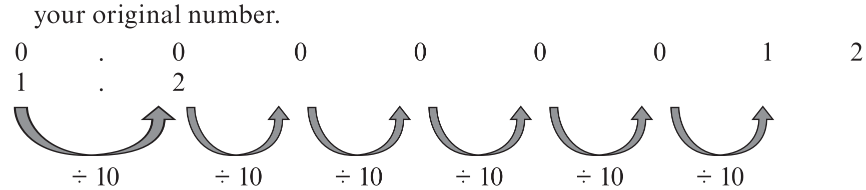
WORKED EXAMPLE 10 (SMALL NUMBERS)

Write 0.000 001 2 in standard form.

Step 1 Write the digits as a number that is greater or equal to one and less than ten.

1.2

Step 2 Work out how many times you have to divide this number by 10 to get back to your original number.



Step 3 Write the number, using the correct power of ten.

$$1.2 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10 = 1.2 \times \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 1.2 \times 10^{-6}$$

Practice question 12

Write these numbers in standard form.

a 0.0034 ....

b 0.000 005 4 ....

c 0.000 507 ....

d 0. 000 000 009 754 ....

Practice question 13

Write these measurements in standard form.

a 0.000 000 000 15 m ....

b 0.003 g ....

c 0.000 000 023 g ....

d 0.0009 m^3 ....

Maths skill 2: Writing numbers to the required number of significant figures (sf)

The rules for **rounding** to a given number of significant figures are similar to those for rounding to a given number of decimal places. The significant figures in a number are counted from the first non-zero digit.

- In 12 345 there are five significant figures, the first is 1.
- In 12 040 there are four significant figures, the first zero (0) is significant, but the final zero is not.
- In 0.012 345 there are also five significant figures, again the first is 1.
- In 0.012 040 there are four significant figures, the first zero (0) is significant, but generally the final zero is not.
- Zeros before the first non-zero digit or after the final non-zero digit are not generally significant, unless, for example, they are necessary to define the accuracy of a number. So 56.012 would be written as 56.0 to 3 sf.

WORKED EXAMPLE 11

Write 124 321 correct to two significant figures.

Step 1 Identify the required number of significant figures.

The first two significant figures are the first and second digits in the number, which have the two highest place values. These are the two digits on the left of the digit to be rounded.

1st sf ↓ *2nd sf* ↓

1 2 4 3 2 1

> **TIP**
> If there is a zero between non-zero digits, for example, 207 224, this counts as a significant figure.

Step 2 Decide whether to round up or down.

Look at the digit in the third significant place.

If it is 0, 1, 2, 3 or 4, leave the first two digits as they are and replace all the rest of the digits in the number with zero.

If it is 5, 6, 7, 8 or 9, increase the digit in the second place by 1 and replace all the rest of the digits in the number with zero.

The next digit is 4 so round down, giving 120 000.

WORKED EXAMPLE 12

Write 0.267 93 correct to two significant figures.

Step 1 Identify the correct number of significant figures.

Identify the first two significant figures. The third is the one to be rounded.

1st sf	2nd sf			
↓	↓			
2	6	7	9	3

Step 2 Decide whether to round up or down.

Look at the digit in the next place.

The next digit is 7 so round up, giving 0.27.

Practice question 14

a There are 423 912 atoms in an amount. Round this to:

i 1 sf ……………………………………….

ii 3 sf ……………………………………….

b Round a mass of 0.324 g to:

i 1 sf ……………………………………….

ii 2 sf ……………………………………….

c There are 1 064 126 atoms in an amount. Round this to:

i 2 sf ……………………………………….

ii 3 sf ……………………………………….

d Round a mass of 0.407 312 g to:

i 1 sf ……………………………………….

ii 2 sf ……………………………………….

Further questions

1 Insert the correct unit prefixes (k, m , μ, c) into each statement.

Each prefix is used only once.

a The diameter of a gold coin is 3 ….. m.

b The thickness of a gold ring is 3 ….. m.

c The mass of a gold bar is 12.4 ….. g.

d The thickness of gold leaf (sheet) is 0.1 ….. m.

2 12 g of carbon is placed in a beaker. This contains 6.02×10^{23} atoms of carbon.

a Another 12 g of carbon are added to the beaker.

Calculate the number of atoms of carbon that are now in the beaker.

Write this answer in standard form.

……

……

b 1.2 g of carbon are added to another beaker.

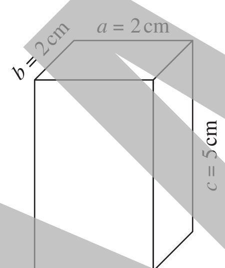
Calculate the number of atoms in this beaker.

Write your answer in standard form.

……

……

3 A student measures a cuboid of aluminium.



a Write each length in metres.

……

……

……

b **i** Calculate the volume of the cube ($a \times b \times c$).

……

ii Write the volume in standard form.

……

The mass of the cube was measured as 0.134 kg.

c **i** Calculate the density of the cube $\left(\frac{\text{mass}}{\text{volume}}\right)$.

....

....

ii Write the density rounded to one significant figure.

....

Chapter 2: Working with data

Why do you need to work with data in chemistry?

Our understanding of chemistry has been developed through the observation of the world around us.

Two types of data may be collected. In chemistry:

- **qualitative data** are often a description of an observation, such as a colour change
- **quantitative data** are based on numbers that have been obtained by measurement.

Data measured on a scale are **continuous data**, but the value of any particular measurement is always an approximation. Measurements always have a level of uncertainty.

Maths focus 1: Collecting data

You can gather data in different ways. Think about a simple experiment in which marble chips are added to hydrochloric acid (see Figure 2.1).

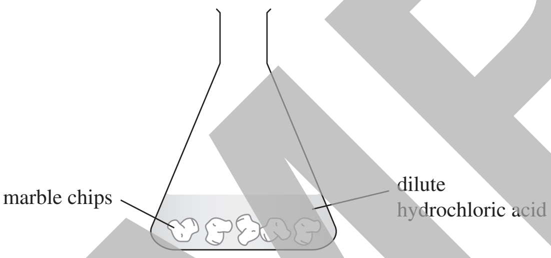


Figure 2.1 The reaction of marble chips with dilute hydrochloric acid

You could make careful observations to create a **qualitative** description of the reaction.

Quantitative data give you more information about the reaction. For example, you could measure the volume of gas produced every 30 seconds (see Figure 2.2). This numerical data can be plotted on a graph and used to find patterns and trends in how fast the reaction takes place.

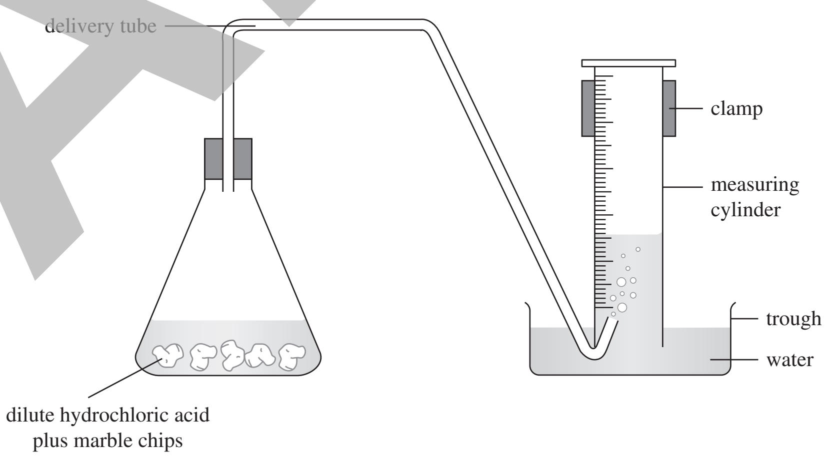


Figure 2.2 Measuring the amount of gas produced

What maths skills do you need to collect data?

1	Reading scales	• Find the level of the liquid • Read the largest number before this level on the scale • Count the small divisions between this and the level of liquid • Add the number of small divisions to the larger number on the scale
2	Recording to the correct number of decimal places	• Write down the reading of the scale division that is exactly at or just before the level when reading from small to large on the scale • Decide whether the level is nearer the whole number or the half-way point between divisions and record the number appropriately

Maths skills practice

How does collecting data help to understand reactions?

- **Digital** measuring equipment, such as a balance, pH meter or temperature probe, displays the measurement directly. The measurements can give you information about a reaction at the beginning and at the end, and even how these quantities change during a reaction.
- **Non-digital** equipment, such as a thermometer, uses a **scale**. A scale is made up of equally spaced divisions with numbers marked at regular intervals. These numbers usually increase in 1s, 2s, 5s or 10s.

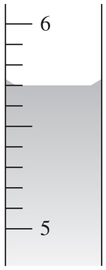
The number of decimal places that you use when recording a measurement is important because it gives information about the level of uncertainty of a measurement. For example, a temperature recorded with no decimal places (such as 24 °C) indicates greater uncertainty than a temperature recorded to one decimal place (such as 24.5 °C).

It is essential that you can read a scale carefully so that you are not adding to the uncertainty of the measurement.

Maths skill 1: Reading scales

WORKED EXAMPLE 1

The diagram shows part of a 10 cm^3 measuring cylinder. What volume of liquid is shown on the scale?



Step 1 Find the level of the liquid.

When reading a volume scale, you should always take the measurement from the bottom of the meniscus, making sure that the surface of the water is at your eye level.

Step 2 Read the *largest* number before this level on the scale.

The largest number on the scale, below the meniscus, is 5.

Step 3 Count the small divisions above this, to the level of liquid.

The meniscus of the liquid is 7 small divisions above 5.

Step 4 Add the number of small division to the larger number.

Work out what each division represents.

There are ten small divisions between 5 and 6 so each division is equal to 0.1 cm^3.

The liquid is 7 small divisions above the 5, so the volume of liquid is: $5 + (7 \times 0.1) = 5.7$

Don't forget to assign the correct units.

5.7 cm^3

> **TIP**
> The surface of water is curved. This is called the **meniscus**.

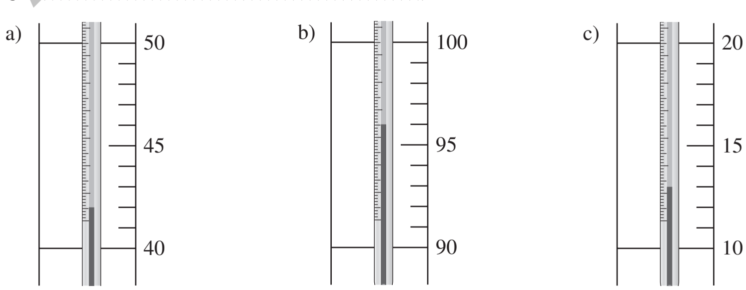
Practice question 1

What temperature is shown on each thermometer?

a ……………………………………….

b ……………………………………….

c ……………………………………….



WATCH OUT
The scales on the measuring cylinders are different.

Practice question 2

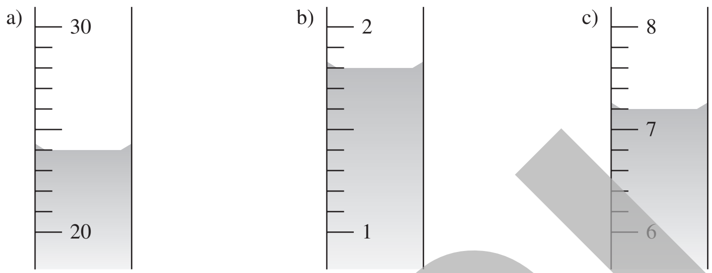
What volume of liquid is there in each measuring cylinder?

(All the measuring cylinders are marked in cubic centimetres, cm^3.)

a ……………………………………….

b ……………………………………….

c ……………………………………….



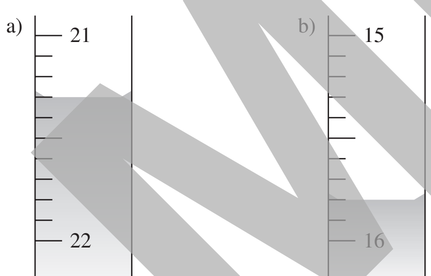
WATCH OUT
The scale on a burette is numbered from top to bottom, so the larger numbers are at the bottom of the scale. You must read it in the opposite direction from a measuring cylinder.

Practice question 3

What volume of liquid is shown in each burette?

a ……………………………………….

b ……………………………………….

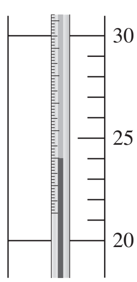


Maths skill 2: Recording to the correct number of decimal places

The more accurate a measurement is, the closer it is to the true value. Using a measuring instrument with greater **resolution** helps to improve **accuracy**. There is less **uncertainty** in measurements taken by an instrument with better resolution.

The resolution of most thermometers is actually the value of half a division, because this is the smallest change that can be measured. For a thermometer with divisions marked every 1 °C, the resolution is 0.5 °C.

You should therefore record measurements from a thermometer correct to one decimal place (for example, 24.0 °C not 24 °C). This shows that there is less uncertainty in the measurement than simply recording the temperature to the nearest whole number (no decimal places).

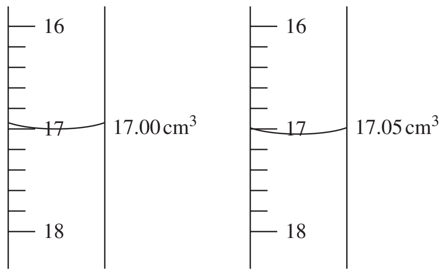


LINK
See Maths skill 3, 'Recording processed data to the correct number of significant figures', p.00.

On a burette the smallest divisions represent 0.1 cm^3.

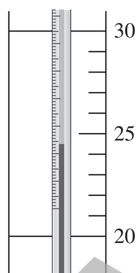
Burettes can also be read to the nearest half division (0.05 cm^3).

The readings from a burette should be recorded to two decimal places.



WORKED EXAMPLE 2

Record the thermometer reading to the correct number of decimal places.



Step 1 Write down the reading of the scale division that is exactly at, or just before, the level when reading from small to large on the scale.

KEY QUESTIONS TO ASK YOURSELF:

- Does the scale read from top to bottom, or bottom to top, of the measuring instrument?

  In this case the scale reads from bottom to top. The numbers increase going up the thermometer.

  On a burette the scale reads from top to bottom. The numbers increase going down the burette.

- Is the scale division just before the level, when reading from small to large on the scale, above, or below the level of the liquid?

  In this case you need read the division that is exactly at or just below the level, 24.

  On a burette you need to read the division that is exactly at or just above the level of the liquid.

Step 2 Decide whether the level is nearer the marked scale division or the half-way point between divisions, and record the number appropriately.

This reading is nearer the half-way point.

This reading should be written:

24.5 °C

WATCH OUT

If it is nearer the marked division, write 0 at the end of the number. Remember to add a decimal point if necessary.

If it is nearer the half-way point between divisions, write 5 at the end of the number. Remember to add a decimal point if necessary.

TIP

Always remember to assign the correct units.

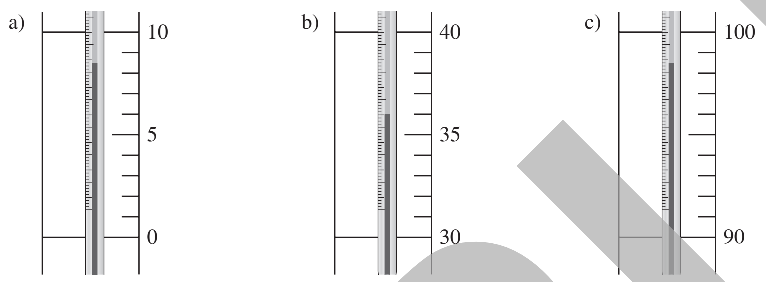
Practice question 4

Write down the temperature shown on each thermometer, to the correct number of decimal places.

a ……………………………………….

b ……………………………………….

c ……………………………………….



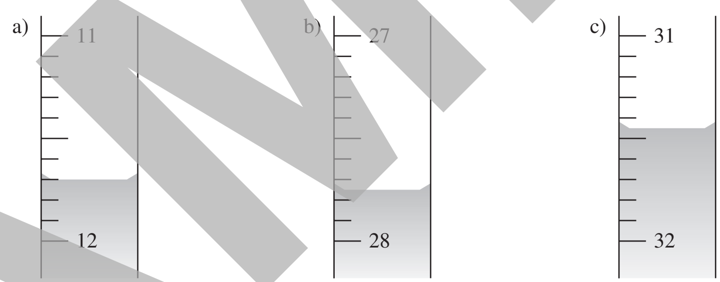
Practice question 5

Write down the volume shown on each burette, to the correct number of decimal places.

a ……………………………………….

b ……………………………………….

c ……………………………………….



Maths focus 2: Understanding types of data

The design of an experiment influences the type of data produced. Usually there will be one **independent variable** that is changed each time, and one **dependent variable** that is measured each time the independent variable is changed.

Often, these variables are numerical (the measurements will be written as numbers). Usually the variable can take on any value, such as when measuring length, or temperature or mass. This is known as **continuous data**.

On other occasions, measurements are made for a range of different categories, for example, types of material. The independent variable that is being changed each time in this case is the type of material. This is not a number and is known as **categorical data**.

Sometimes an independent variable can only take on certain values. In this case it is known as **discrete data**. This type of data is less common in chemistry.

LINK
See Chapter 3, 'Drawing charts and graphs'.

LINK
See Chapter 3, Maths Focus 3, 'Drawing line graphs'.

WATCH OUT
For quantitative data, the dependent variable will always be numerical (either continuous or discrete) but the independent variable may not be.

LINK
See Chapter 3, Maths Focus 1, 'Drawing bar charts'.

TIP
If a measurement is made at regular time intervals, then the independent variable is time.

TIP
The independent variable does not have to be a number. It could be the type of metal.

What maths skills do you need to understand different types of data?

1 Identifying the independent and dependent variables	• Find the variable that was changed during the experiment (the independent variable) • Find the variable that was measured each time (the dependent variable)
2 Distinguishing categorical, continuous and discrete data	• Decide whether the data are recorded as words (categorical data) or numbers • Decide whether numerical data can take on any value

Maths skills practice

How does understanding different types of data help to decide what type of graph to draw?

There are different types of data:

- Some data can be sorted into categories (groups) but the categories cannot be easily ordered, for example, the names of materials. This is known as *categorical* data.
- Sometimes the numerical values can take any value within a certain range, for example, the temperature of an object. This is *continuous* data.
- *Discrete* data occur where the values can only take certain values, for example, the numbers of protons in an atom can only be whole numbers.

If the independent variable is categorical, present it on a bar chart. Label the categories of the independent variable along the horizontal axis.

Always represent continuous data in a line graph, because the points between the plotted data also have values.

The independent variable should always be plotted on the horizontal axis, and the dependent variable on the vertical axis.

Maths skill 1: Identifying the independent and dependent variables

WORKED EXAMPLE 3

Read the experiment description. Write down the independent variable and the dependent variable.

A student adds some marble chips to hydrochloric acid and measures the temperature every 30 seconds for 5 minutes.

Step 1 Identify the variable that is changed each time (independent variable).

The student measures the temperature every 30 seconds, so the independent variable is time.

Step 2 Identify the variable that is measured each time the independent variable is changed. (the dependent variable).

The student measures the temperature every 30 seconds, so the dependent variable is temperature.

> **TIP**
> Sometimes a description of an experiment gives the units but not the name of the variable. You need to remember that measuring in cubic centimetres (cm^3) means that the variable is *volume* or that measuring in grams (g) means that the variable is *mass*.

Practice question 6

A student carries out four different experiments. Read the experiment description and write down the independent and dependent variables for each.

a A student adds 10 cm^3 of acid, 1 cm^3 at a time, to a beaker containing an alkali. She uses a pH meter to measure the pH each time.

....

....

b She then carries out another experiment and adds some marble chips to a flask containing acid. She measures the mass every 30 seconds for 5 minutes.

....

....

c For her third experiment the student adds sodium thiosulfate to hydrochloric acid and times how long it takes for the solution to turn cloudy. She repeats this at four different temperatures.

....

....

d Finally the student uses pH paper to test five different types of substance.

....

....

Practice question 7

A scientist finds the temperatures at which elements with different atomic numbers melt. In this experiment what is:

a the independent variable

....

b the dependent variable?

....

Maths skill 2: Distinguishing categorical, continuous and discrete data

WORKED EXAMPLE 4

Read the experiment description. Write down whether the independent variable is categorical, continuous or discrete.

A student adds some marble chips to hydrochloric acid and measures the temperature every 30 seconds for 5 minutes.

Step 1 Decide whether the independent variable is recorded in words or numbers.

If it is recorded in words, then the data is categorical.

In this case the data is numerical so a further decision needs to be made.

Step 2 Decide whether the numerical data can take on any value.

The independent variable is time. This is recorded in numerals and can take on any value. This is because, even though the student has chosen to measure every 30 seconds, the values in between have meaning.

The independent variable is therefore continuous.

(If numerical data cannot take on any value it is discrete data.)

Practice question 8

Read the experiment descriptions in Practice questions 6 and 7 again. Write down whether each independent variable is categorical, continuous or discrete.

....

Maths focus 3: Recording and processing data

Data alone cannot help you to understand chemistry. Recording data helps to communicate measurements clearly to other people. It also helps you to process the data, for example, using maths to calculate the mean and other quantities.

Recording data clearly in a table makes it easier to plot a graph, and this helps you to identify any patterns or trends in the data.

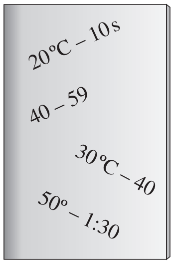
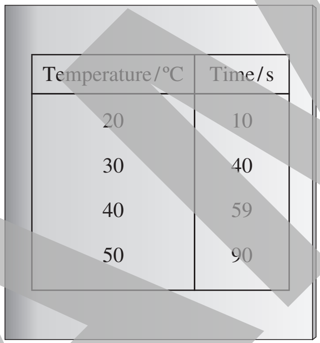


Figure 2.3 Unorganised data

Figure 2.3 shows data recorded in an unorganised way. The units are not used consistently and the measurements are not presented in order.



Temperature / °C	Time / s
20	10
30	40
40	59
50	90

Figure 2.4 Organised data

In this example the data is recorded in a table. The units are the same throughout the experiment and the measurements are presented in a systematic way. This makes it easy to spot a pattern in the data. As the temperature increases, the time increases.

What maths skills do you need to record and process data?

1 Drawing tables	• Work out how many columns and rows you need • Draw up the table • Add headings to each column • Add the values of the independent variable
2 Drawing tables to help process data	• Work out how many extra columns you need • Draw the table, as described in Maths skill 1

3	Recording processed data to an appropriate number of significant figures	• Use a calculator to complete any calculations • Note the smallest number of significant figures that occur in the data • Round the calculated values to this number of significant figures

Maths skills practice

How does recording and processing data help understand chemistry?

A clear table helps to present information about different substances so that their properties can be compared.

A table also helps to organise data making it easier to plot a graph which makes it easier to identify patterns and trends.

Processing data to work out the mean helps you to ensure there is less uncertainty in the data.

Maths skill 1: Drawing tables

Record the independent variable in the left-hand column of a table and the dependent variable in the right-hand column.

For categorical data, add words to the table in the left-hand column and add any numerical data for the dependent variable in the right-hand column.

For continuous and discrete data the independent variable is numerical. The values can be written in the left-hand column before the experiment begins.

You can also use a table to record qualitative data. In this case, record your observations in the right-hand column.

WORKED EXAMPLE 5

A student pours 20 ml of hydrochloric acid into a polystyrene cup and measures its temperature.

She then adds a small piece of magnesium ribbon.

The student takes the temperature every 10 seconds for one minute.

Step 1 Work out how many columns and how many rows she needs.

She needs two columns, one for the independent variable and one for the dependent variable.

The student measures the temperature at 0, 10, 20, 30, 40, 50 and 60 seconds (1 minute), so she needs a header row plus 7 more rows in the table.

Step 2 Draw up the table.

Step 3 Add the headings to each column.

The independent variable is time, with its unit s (for seconds), so the left-hand column heading is 'Time/s'.

The dependent variable is temperature, with its unit °C (for degrees Celsius), so the right-hand column heading is 'Temperature/°C'.

> **WATCH OUT**
> Always remember to separate the name of the variable and its unit symbol with a /. This can be called a slash, slant, solidus or stroke.

Step 4 Add the values of the independent variable to the left-hand column.

The values of the independent variable are written out before the experiment starts.

Time/s
0
10
20
30
40
50
60

> **TIP**
> Adding the unit symbol to the column headings means that you don't have to write it after each number.

Practice question 9

Draw a table that could be used for each experiment.

a A student adds 6 cm^3 of acid, 1 cm^3 at a time to a beaker containing an alkali. He uses a pH meter to measure the pH each time.

b A student adds some marble chips to a flask containing acid. She measures the mass in grams (g) every 30 seconds for 3 minutes.

> **TIP**
> pH has no units because it is a logarithm, and this is beyond the requirements of this book.

c A student adds sodium thiosulfate to hydrochloric acid (10 °C) and times, in seconds, how long it takes for the solution to turn cloudy. She repeats this at 20 °C, 30 °C and 40 °C.

d A student uses pH paper to test five different types of substance, A–E.

Maths skill 2: Drawing tables to help process data

Sometimes you need to apply mathematical processing to the data you collect, for example:

- You may need to calculate the mean for each set of repeated data.
- You may have measured the mass and volume of different metals in order to calculate the density.

Recording both the raw data and processed data in a well-organised table can help to make calculations easier.

WORKED EXAMPLE 6

A student times how long it takes 0.5 g of small marble chips to react with 25 cm^3 of acid. He repeats the experiment two more times until he has a set of three measurements.

He then carries out three experiments with medium marble chips, and another three experiments with large marble chips.

Draw up the table that he could use to record these results and calculate the mean.

Step 1 Work out how many extra columns are needed.

The student needs one column for the independent variable (time in seconds) plus three columns for the dependent variable, because he needs to include the results of the three experiments for each chip size. He also needs an extra column to record the mean that he calculates.

In total the student therefore needs five columns in his table.

Step 2 Draw a table, as described in Maths skill 1.

Work out how many rows the student needs.

The student is testing thee sizes of marble chip so after the header row he needs three more rows.

Add the headings to each column.

The independent variable is size of the marble chips. This does not have a unit.

There are three columns for the dependent variable (Time/s). This needs one overall heading above and three separate column headings: Test 1, Test 2 Test 3 below.

Time/s		
Test 1	**Test 2**	**Test 3**

The column on the right of the table should have the heading 'Mean', with the correct unit symbol (the same as the dependent variable) of s for seconds.

Add the values of the independent variable that are going to be tested to the left-hand column.

In this case the independent variable is categorical, so the words 'small', 'medium' and 'large' need to be added.

Size of marble chip	Time/s			Mean/s
	Test 1	Test 2	Test 3	

Practice question 10

Draw up the table for each student's experiment, described below.

a A student adds sodium thiosulfate solution to hydrochloric acid at 20 °C and times how long it takes for the solution to turn cloudy. He carries out the experiment two more times. He then carries out the experiment three times at 30 °C and three times at 40 °C.

> **TIP**
> The order of columns can help with calculations. Put the columns in the order that the numbers will be used in the calculation.

b Another student is finding out the density of four pieces of metal (copper, iron aluminium and tin). She finds the volume and mass of each piece of metal and records these in a table. She then works out the density using the formula:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Maths skill 3: Recording processed data to the correct number of significant figures

> **LINK**
> See Chapter 1 for more on significant figures.

When you carry out a calculation with experimental data, the number of significant figures in your answer should be the same as the smallest number of significant figures used in the data values. Your calculated answer should not imply more accuracy than occurs in the original measurements.

However, there are other factors that can result in uncertainty. Random errors often occur when you are carrying out an experiment. Working out the mean of repeated measurements reduces uncertainty due to random errors.

Repeat results that are close together indicate that the data is precise. However, even if results are in close agreement this does not necessarily mean that the data values are accurate. There could be a systematic error that is making all the data slightly too large or too small. An example of a systematic error could be a zeroing error on a balance.

LINK

See Chapter 1, Maths focus 3, Maths skill 2, 'Writing numbers to the correct number of significant figures (sf)'.

WATCH OUT

If you have more than one set of measurements (for example, mass and volume), remember to use the number of significant figures in the data value that has the smallest number of significant figures.

TIP

A reading that is very different to the others it is called an **outlier**. If you can explain why the measurement is so different (for example, due to an error in the experiment) then it should be ignored when calculating the mean.

WORKED EXAMPLE 7

A student measures the volume of carbon dioxide produced during a reaction.

Calculate the mean volume of the three measurements. Record your answer to an appropriate number of significant figures.

Volume/cm³		
Test 1	**Test 2**	**Test 3**
22.4	22.2	21.8

Step 1 Use a calculator to complete any calculations.

Calculate the mean:

$$\frac{22.4 + 22.2 + 21.8}{3} = 22.1333$$

Step 2 Note the lowest number of significant figures in the data.

Each measurement has three significant figures.

Step 3 Round the calculated values to this number of significant figures.

22.1 cm³

Practice question 11

a A student uses a burette to measure the volume of acid required to neutralise an alkali. She carried out the experiment three times. Calculate the mean result of the three experiments. Record the mean to an appropriate number of significant figures.

Volume/cm³		
Test 1	**Test 2**	**Test 3**
20.05	20.10	19.95

....

b A student times how long it takes for a reaction to form 20 cm³ of hydrogen gas.

It takes 16 seconds.

Calculate the mean rate of reaction: $\frac{\text{volume}}{\text{time}}$

....

c A student measured how much the mass decreased during a reaction during 10 s. The loss of mass was 1.24 g. Calculate the mean rate of reaction: $\frac{\text{mass}}{\text{time}}$

....

Further questions

1 A student investigated the density of three lumps of metal.

First she used a balance to find the mass of each lump of metal.

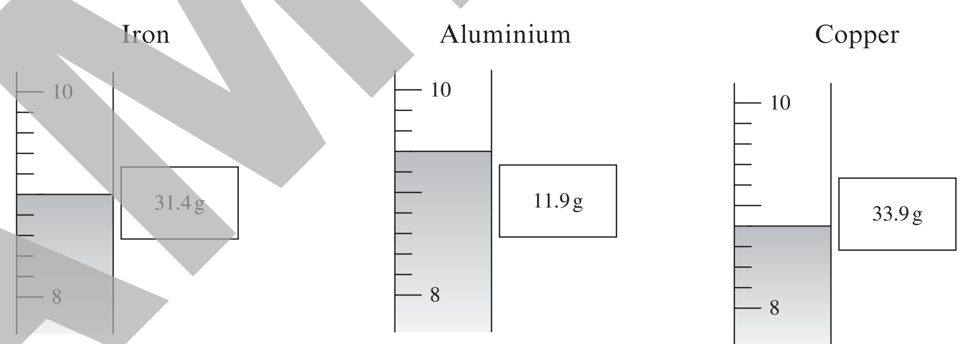
Then she added 5 cm^3 of water to a measuring cylinder. She then dropped a lump of iron into the measuring cylinder and noted the new volume.

The change in volume was equal to the volume of the lump iron.

She recorded the mass and new volume in a table. She then calculated the volume of each lump and the density of each.

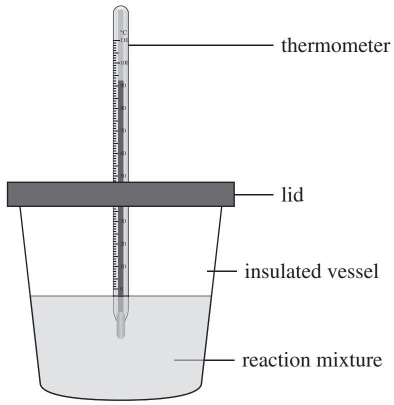
a Draw up a table in which the student could record her measurements and calculations.

The measuring cylinder and digital balance readings are shown below. Record the measurements in the table.



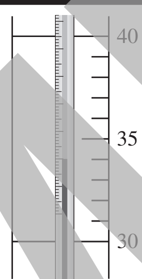
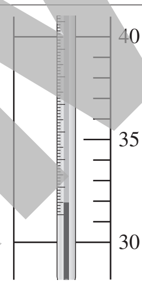
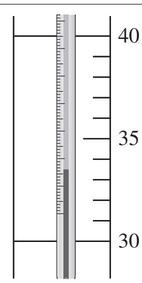
b Calculate the density of each metal and record in the table to an appropriate number of significant figures.

2 The temperature was measured during two different reactions. The temperature of the solution at the start was 24 °C. Each experiment was carried out three times.



Reaction 1: Copper sulfate solution + zinc powder

a Record the maximum temperature measurements in the table below.

Experiment	Thermometer diagram	Maximum temperature of solution/°C	Temperature change/ degrees
1			
2			
3			

b The temperature of the solution was 23 °C at the start. Calculate the temperature change for each experiment and record this in the table.

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c Calculate the mean change for the copper sulfate + zinc reaction, giving your answer to an appropriate number of significant figures.

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Reaction 2: Potassium nitrate + water

Test	Maximum temperature of solution/°C	Temperature change/°C
1	20.5	–2.5
2	20.0	–3.0
3	19.0	–4.0

d Calculate the mean change for the potassium nitrate + water reaction to an appropriate number of significant figures.

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e State whether each reaction is exothermic (energy is transferred to the surroundings) or endothermic (energy is transferred from the surroundings).

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